Isgur-Wise functions for confined light quarks in a colour electric potential*

H. HØGAASEN†
Department of Physics, University of Oslo,
N-0316 Oslo, Norway.

and

M. SADZIKOWSKI‡
Institute of Physics, Jagellonian University,
ul. Reymonta 4, PL-30 059 Kraków, Poland.

Abstract

We explore the influence on the Isgur-Wise function of the colour electric potential between heavy and light quarks in mesons. It is shown that in bag models, its inclusion tends to restore light quark flavour symmetry relative to the MIT bag predictions, and that relative to this model it flattens the Isgur-Wise function. Results compare very well with observations.

*Work supported in part (M.S.) by the KBN grant no. 20-38-09101. We also thank the Norwegian Research Council for a travel grant.

†E-mail HOGAASEN@VUOEP6.UIO.NO
‡E-mail UFSADZIK@ZTC386A.IF.UJ.EDU.PL

The physics of states containing a heavy quark is the subject of much research. This is evident because in the limit where the mass of the heavy quark grows beyond any limit, the heavy quark can be regarded as a static source where the heavy quark spin effectively decouples from the interaction. The degrees of freedom of the light quark(s) then become the main dynamical variables, and the system is a fine theoretical laboratory to study properties of bound light quarks. It is hoped that the properties one can deduce for such systems is of relevance for physical systems where the heavy quark is a b or a c quark.

One of the most brilliant applications of the Heavy Quark Symmetries (see for example [1] and citation therein) is the description of semileptonic decays of particles containing a heavy quark in terms of only one unknown quantity the Isgur - Wise function (IW) [2]. The dependence of this function of light degrees of freedom is very complicated, and establishes an essential problem in all calculations that leads to numerical values of, not only branching ratios of semileptonic decays, but also in part of two-body decays branching ratios and determinations of the element $|V_{cb}|$ of Kobayashi-Maskawa matrix.

A reliable calculation of the IW function based on first principles is beyond our power up to now, so a variety of different models must be used to estimate some numerical results. One of the possible approaches in the calculation of the IW function was based on the MIT bag model [3]. Bag models were successful in describing several important features connected with particles containing one heavy quark (for example in papers [4], [5], [6], [7]). In this paper we deal with the Coulomb-like bag model, a very natural candidate for reasonable description of heavy - light quark system.

In the infinite mass limit the Heavy Quark Effective Theory ([1]) predicts that the IW function is related to the overlap of the light degree of freedom wavefunction multiplied by a kinematical factor. In the case of the ground state to ground state transitions we have [8][1]:

$$\xi(\omega) = \sqrt{\frac{2}{\omega+1}} < \Phi_{Q'l} | \Phi_{Ql} > \tag{1}$$

where the ket in above equation describes the light degree of freedom wavefunction of the parent (Φ_{Ql}) and the bra of the daughter ($\Phi_{Q'l}$) particle. The

[1]In case of Λ baryons the IW function is often defined in literature also as a pure overlap, without kinematical factor.

IW function is independent of the spin of the heavy quark. Before we come to our main topic we make some general remarks about the IW function valid for all models satisfying the following two assumptions:

- The valence quark approximation
- Spherical symmetry of the ground state probability density

Due to the first assumption the wavefunction of the light quark in the particle rest frame can always be written in the form:

$$\Phi^0(x) = \Phi^0(\vec{x})e^{-iEt} \tag{2}$$

where $\Phi^0(\vec{x})$ is a bispinor field and E is its ground state energy.
The second assumption states that the product:

$$\rho(r) \equiv \Phi^{0\dagger}(x)\Phi^0(x) \tag{3}$$

is independent of angles.

Consider the IW function for mesons. In the decay of one particle to another one can always choose the frame where two particles move along the z-axis with equal and opposite velocities (modified Breit frame [9]). In this frame the overlap takes the form:

$$< \Phi_{Q'l}|\Phi_{Ql} >= \int_{CK} d^3x \Phi^\dagger_{Q'l}(x)\Phi_{Ql}(x)|_{t=0} \tag{4}$$

where:

$$\Phi_{Ql}(x) = S(-\vec{v})\Phi^0_{Ql}(x,y,\gamma z)e^{-iE\gamma vz} \tag{5}$$

is a wavefunction describing the light quark in the parent particle that is moving with velocity $v = |\vec{v}|$ along the negative z-axis.

$$\Phi_{Q'l}(x) = S(\vec{v})\Phi^0_{Q'l}(x,y,\gamma z)e^{iE\gamma vz} \tag{6}$$

is a wavefunction describing the light quark of the daughter particle which is moving with velocity v in positive direction along the z-axis.
Both functions are related to the wavefunction in their rest frames via boost operator $S(\vec{v})$ and Lorentz transformations:

$$L^{-1}_{\pm\vec{v}}(0,\vec{x}) = (\mp\gamma vz, x, y, \gamma z) \tag{7}$$

We define the volume CK as the region explored by both wavefunctions in the moment of the decay. In case of the MIT bag model it is the intersection of contracted bags [3], in case of other models it may be the "infinite" region. Lets also note, that according to Heavy Quark Symmetries the wavefunctions $\Phi^0_{Q'l}$ and Φ^0_{Ql} are the same.

If one writes down the overlap in terms of the wavefunctions from their rest frames, then in the integral over space there will always appear a product of boost matrices: $S^\dagger(\vec{v})S(-\vec{v})$. Using the identity between the velocity $|\vec{v}|$ and scalar product of velocities of both particles $\omega = vv'$ valid in the "Breit frame":

$$|\vec{v}| = \sqrt{\frac{\omega - 1}{\omega + 1}} \tag{8}$$

one can write:

$$S^\dagger(\vec{v})S(-\vec{v}) = \sum_{n=0}^{\infty} \hat{\alpha}_n \left(\frac{\omega - 1}{\omega + 1}\right)^n \tag{9}$$

where only the first operator $\hat{\alpha}_0$ is known. It is equal to unity. Using (1), (4), (5), (6) and (9) and rescaling the z - variable by a factor γ one gets:

$$\begin{aligned} \xi(\omega) = \left(\frac{2}{\omega + 1}\right)\Big[\int_{K(0,R)} d^3x \rho(r) j_0\Big(2E\sqrt{\frac{\omega - 1}{\omega + 1}}r\Big) + \\ + \sum_{n=1}^{\infty}\left(\frac{\omega - 1}{\omega + 1}\right)^n \int_{K(0,R)} d^3x \Phi^{0\dagger}(\vec{x})\hat{\alpha}_n \Phi^0(\vec{x}) e^{-2iEvz}\Big] \end{aligned} \tag{10}$$

where $K(0,R)$ is the region explored by the light degree of freedom wavefunction in the rest frame of particle. For the simplest bag models it is a spherical bag but for other models it can be an "infinite" region. j_0 is the usual spherical Bessel function of order zero. Now it is straightforward to find the slope parameter:

$$\rho^2 = -\frac{d\xi}{d\omega}|_{\omega=1} = \frac{1}{2} + \frac{1}{3}E^2 < r^2 > -\frac{1}{2}\int_{K(0,R)} d^3x \Phi^{0\dagger}(\vec{x})\hat{\alpha}_1 \Phi^0(\vec{x}) \tag{11}$$

Unfortunately these results are not giving us enough information. The additional assumption that $S(\vec{v})$ is hermitian (for example true for the free boost) leads to the following formula:

$$\hat{\alpha}_n(r) = 0 \qquad for \quad n = 1, 2, ... \tag{12}$$

and consequently:

$$\xi(\omega) = (\frac{2}{\omega+1}) \int_{K(0,R)} d^3x \rho(r) j_0(2E\sqrt{\frac{\omega-1}{\omega+1}}r) \tag{13}$$

$$\rho^2 = \frac{1}{2} + \frac{1}{3}E^2 < r^2 > \tag{14}$$

These relations were found in paper [3]. The above results have a practical meaning for all models in which the density and the ground state energy of the light degrees of freedom are known.

It was shown earlier by K. Zalewski and one of us [3], that in the case of mesons the MIT bag model wave functions for light quarks gave results for the IW function that compared very well with what we believe is known experimentally. When we speak of the MIT bag model here we speak of its simplest tractable version, the static approximation where the bag is spherical in its rest frame. The confinement mechanism then correspond to a world scalar potential which is zero inside and infinite outside the bag surface. There is no four vector like potential between the "infinitely" heavy quark, which defines the center of the bag, and the light quark. Up to maximal distance the bag radius, the light quark satisfies the free particle Dirac equation. One certainly would believe that there is an influence of the gluonic interaction at intermediate distances, theoretically one would expect something like a Coulomb-like potential, modified at very small distances by asymptotic freedom effects. This certainly is the result from lattice gauge calculations of the potential between two heavy quarks. It is therefore of some interest to find out how such an inner potential influences the successful prediction of the IW function that was obtained from the simplest (MIT) bag wave functions for the light quark. We therefore model the interaction of the heavy and light quark with a Coulomb potential for $r < R$, solve the Dirac equation there and quantize the quark energy with the MIT-Bogoliubov boundary condition at $r = R$ [10]. We normalize the potential so that it is zero at the boundary. We shall comment later on the influence of asymptotic freedom effects at very small r. The inner potential is now of the form:

$$V(r) = -b\left(\frac{1}{r} - \frac{1}{R}\right) \tag{15}$$

We now define the light quark energy E as solution of the Dirac equation:

$$H\Phi^0(r) = E\Phi^0(r) \tag{16}$$

where:

$$H = \vec{\alpha}\vec{p} + \beta m + V(r) \tag{17}$$

Here m is the mass of the light quark. The quark energy is determined from the boundary condition at the bag radius R:

$$i\vec{\gamma}\hat{r}\Phi^0(R) = \Phi^0(R) \tag{18}$$

where $\hat{r}$ is a radial unit vector. The light quarks wave function and its energy are evidently functions of the strength of the central Coulomb potential characterized by the constant b. This is of course not a parameter that we know the size of. For illustrative purposes we shall use the value obtained from a fit to charmed mesons in [6], $b = 0.542$ and their corresponding bag radii for strange and nonstrange quarks. This value of b correspond to a strong coupling constant $\alpha_s = 0.407$, a quite reasonable value. All parameters now fixed, we easily compute the IW function from formula (13).

In fig. 1, we show the IW function for massless light quarks in the cases $b = 0$ (MIT bag [3]), $b = 0.542$ [6] and the ARGUS data [11]. To compare the IW function with the data we fit the Cabbibo-Kobayashi-Maskawa matrix parameter $|V_{cb}| = 0.0386\sqrt{\tau_B/1.29ps}$. The χ^2 of this fit is 9.3 for 7 degrees of freedom. As the IW functions we used the approximation of the exact result by the functions:

$$\xi^B(\omega) = (\frac{2}{\omega+1})^{1.52+\frac{0.45}{\omega}} \tag{19}$$

for $\bar{B} \to De\bar{\nu}_e$ and

$$\xi^{B_s}(\omega) = (\frac{2}{\omega+1})^{1.67+\frac{0.59}{\omega}} \tag{20}$$

for $\bar{B}_s \to D_s e\bar{\nu}_e$ similar for ones used in [3]. The appropriate results for slope parameters are:

$$\rho_B^2 = 0.98 \qquad \rho_{B_s}^2 = 1.135 \tag{21}$$

The slope parameter for $\bar{B} \to De\bar{\nu}_e$ is in a good agreement with what one finds in lattice calculations. One group [12] found $\rho_B^2 = 1.0(8)$, another [13] $\rho_B^2 = 1.2(+7-3)$.

As can be seen from the figure, the differences between the free MIT bag and Coulomb-like bag are not very big, but the central potential clearly has led to a flatter IW function.

Considerable insight can be gained by looking at the expression for the derivative of the IW function at minimum recoil, i.e. at $\omega = 1$ given in formula (14), where the product X of the quark energy and the root–mean square radius of the light quark probability distribution enters in an essential manner:

$$X = E\sqrt{< r^2 >} \tag{22}$$

In the region of interest to us this product is given to a very good approximation by a linear fit $X = a_1 + a_2 mR$ where the coefficients a_1 and a_2 decreases with increasing strength of the Coulomb potential b. For $b = 0$ we find:

$$X = 1.49 + 0.29mR \tag{23}$$

for $b = 0.542$:

$$X = 1.15 + 0.17mR \tag{24}$$

This has interesting consequences that will show up in a moment, when we compute decay probabilities: The Coloumb potential tend to restore flavour symmetry in decay probabilities.

Using the formulas for decay rates (for example from [3]) and the functions (19), (20) we get:

$$Br(B \to Dl\bar{\nu}) = 1.79(\tau_B/1.29ps)\% \tag{25}$$

$$Br(B \to D^* l\bar{\nu}) = 5.05(\tau_B/1.29ps)\% \tag{26}$$

$$Br(B_s \to D_s l\bar{\nu}) = 1.74(\tau_{B_s}/1.29ps)\% \tag{27}$$

$$Br(B_s \to D_s^* l\bar{\nu}) = 5.02(\tau_{B_s}/1.29ps)\% \tag{28}$$

Agreement with available data $(1.7 \pm 0.4\%)$ and $(4.8 \pm 0.6\%)$ [14] for B to D and B to D^* transitions is quite satisfactory. What is more interesting we also found that:

$$Br(B \to Dl\bar{\nu}) \approx Br(B_s \to D_s l\bar{\nu}) \tag{29}$$

$$Br(B \to D^* l\bar{\nu}) \approx Br(B_s \to D_s^* l\bar{\nu}) \tag{30}$$

as should be expected from naive observation based on $SU(3)$ flavour symmetry. This result was absent in case of the MIT bag. It can be explained

by the fact that the increase of the energy of the ground state with the increasing light quark mass is much better compensated by the decrease of the mean square radius in case of the Coulomb-like than in the MIT bag. This is the source of the difference between (23) and (24). The most important thing, however, is that the product X is smaller in the Coulomb bag than in the MIT bag so that the slope given by eq (14) does not vary as much with varying light quark mass.

Measuring and comparing branching ratios to test light quark flavour symmetry is therefore extremely interesting.

The sceptical reader might be worried by the fact that the Coulomb potential does not incorporate asymptotic freedom, one of the most celebrated properties of gluonic interaction. There are all reasons to check, whether the singularity of the Coulomb potential at the origin distorts our results. This is not the case however: We integrated the Dirac equation with the potential $V(r)$ of equation (15) for $r > r_0$ and constant potential $V(r_0)$ for $r < r_0$. With $r_0 = 0.2\ GeV^{-1}$ there were no differences on the 1% level for the calculated IW-function relative to the values we found with the pure Coulomb potential.

Acknowledgements
We thank Kacper Zalewski for a useful discussion.

References

[1] M. Neubert, SLAC-PUB-6263 (1993).

[2] N. Isgur and M. B. Wise, Phys. Lett. **B232** (1989)113; **B237** (1990)527.

[3] M. Sadzikowski and K. Zalewski, Z. Phys. **C59** (1993)677.

[4] E. V. Shuryak, Phys. Lett. **B93** (1980)134.

[5] D. Izatt, C. De Tar and M. Stephenson, Nucl. Phys. **B199** (1982)269.

[6] W. Wilcox, O. V. Maxwell and K. A. Milton, Phys. Rev. **D31** (1985)1081.

[7] M. Sadzikowski, Acta Phys. Pol. **B24** (1993)1121.

[8] K. Zalewski, Phys. Lett. **B264** (1991) 432.

[9] O. Lie-Svendsen and H. H$\o$gaasen, Z. Phys. **C35** (1987) 239.

[10] H. H$\o$gaasen, J. M. Richard and P. Sorba, Phys. Lett. **119B** (1982)272.

[11] H. Albrecht et al. (ARGUS Collaboration), Z. Phys. **C 57** (1993)533.

[12] C. W. Bernard, Y. Shen and A. Soni, Nucl. Phys. (Proc. Suppl.) **B 30** (1993)473.

[13] S. P. Booth et al. (UKQCD Collaboration), Phys. Rev. Lett. **72** (1994)462.

[14] Particle Data Group, Phys. Rev. **D45** (1992) Part 2.